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We constructed a simple model of lung distensibility in which the relationship between pressure (P) and volume (V) is of the exponential form obtained empirically: $V = a + be^{cP}$, where a, b, and c are constants, and e is the base of the natural logarithm. The model lung is idealized as a frictionless diaphragm moving in a cylinder. Diaphragm movement is impeded by a variable force (F). By assuming that the rate of change of F is proportional to the inverse of the distance of the diaphragm from a maximal distance, the pressure-volume relationship assumes the above exponential form. The model was applied to 14 randomly selected patients, seven with increased lung compliance and seven with decreased lung compliance. For all but three of the patients, the model accounted for over 99% of the variability of the data; for all patients this value was greater than 96%. Correlations were calculated among both traditional measures of lung elasticity and parameters introduced by construction of the model. In general, the various measures correlated poorly with one another. It is argued that the concept of "lung elasticity" will remain imprecisely defined until the relationship between lung structure and lung function is better understood. (Henry Ford Hosp Med J 1988;36:44-7)

Salazar and Knowles (1) observed that the relationship between lung pressure (P) and lung volume (V), for static lung volumes greater than the functional residual capacity (FRC), closely fits an exponential function of the form:

$$V = a + be^{cP} \tag{1}$$

where e is the base of the natural logarithm, and a, b, and c are constants. This exponential relationship has been confirmed by other laboratories (2-4).

Traditional parameters of lung elasticity include the elasticity at FRC (E*FRC), the maximal static recoil pressure (MSRP), and the coefficient of retraction (CR) (5). In addition, the work of Gibson et al (2) suggests that the negative of exponent c [see equation (1)] should prove of value in identifying abnormal lung elasticity.

To what extent do any or all of these parameters represent the property of "lung elasticity"? The answer awaits an understanding of how the elastic components of the lung are structurally integrated to yield the pressure-volume relationship described by Eq (1). We underscore this point by constructing a crude model of lung distensibility in which the pressure-volume relationship assumes this exponential form. Within the context of our model, "lung elasticity" cannot be expressed in terms of a single parameter. Furthermore, we find that the aforementioned parameters correlate poorly with one another and hence cannot all represent the same property.

Model Description

The lung is idealized as a frictionless diaphragm with area equal to unity moving in a cylinder (Figure). Diaphragm movement is impeded by a variable force (F). The assumption is made that the rate of change of F is proportional to the inverse of the distance of the diaphragm from a maximal distance X(MAX), that is:

$$F' = dF/dX = R/[X(MAX) - X] \tag{2}$$

where R is the proportionality constant. Integration of this expression yields:

$$F = -R \ln[X(MAX) - X] + E \tag{3}$$

where E is a constant. Observe that E = F when X = X(MAX) − 1. Since the area of the diaphragm is unity, X = V, and F = P; therefore:

$$P = -R \ln[V(MAX) - V] + E \tag{4}$$

This can be reexpressed:

$$V = V(MAX) - e^{(E/R)}e^{(-P/R)} \tag{5}$$

If the constants a, b, and c are defined as follows:

$$a = V(MAX) \tag{6}$$

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$$b = -e^{(E/R)} \quad (7)$$

$$c = -1/R \quad (8)$$

then Eq (5) is found to be identical to Eq (1). Hence, the pressure-volume relationship for the model lung is shown to be of the same form as the empirically obtained pressure-volume relationship.

Expressions for E and R in terms of a, b, and c can be derived from Eq (7) and (8):

$$R = -1/c \quad (9)$$

$$E = -\ln(-b)/c \quad (10)$$

If we define X(0) as the diaphragm position in which diaphragm movement is not impeded, that is, when $F = 0$, then from Eq (3) we find:

$$X(0) = X(MAX) - e^{(E/R)} \quad (11)$$

If we further define $F'(0)$ as F' when $X = X(0)$, then Eq (2) and (11) yield:

$$F'(0) = R/e^{(E/R)} \quad (12)$$

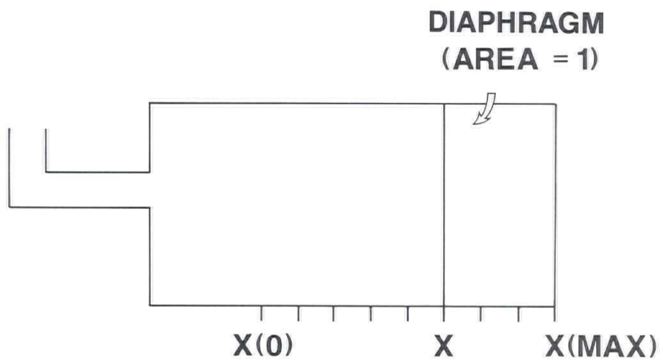
$F'(0)$ is analagous to the force constant of an "ideal" spring. In the case of an ideal spring, F' remains constant as the spring is stretched. In the present model, however, F' varies as a function of diaphragm position.

E*FRC is the slope of the pressure-volume curve when V = FRC. Therefore, Eq (2) can also be used to calculate E*FRC:

$$E{*}FRC = F'(FRC) = R/[V(MAX) - V(FRC)] \quad (13)$$

Methods

Pressure-volume data from seven patients with decreased lung compliance and seven patients with increased lung compliance were analyzed. A patient was defined as having decreased lung compliance if expiratory compliance at FRC was more than two standard deviations below the mean compliance for normal subjects with the same age and lung volume (6). Increased compliance was similarly defined as more than two standard deviations above the mean. Patient data were selected in a random fashion from the set of all pressure-volume curves determined by our laboratory between 1980 and 1983. For each subject, subdivisions of lung volume were measured in a constant-volume body plethysmograph (7). The pressure-volume data were obtained using an esophageal balloon as previously described (8), with the subject seated in a variable-volume plethysmograph (9). Functional residual capacity was identified during a period of tidal breathing. After a series of three full inspirations, the subject held his/her breath at total lung capacity with an open glottis while MSRP was measured. The subject then relaxed against a closed airway and subsequently was allowed to exhale in a stepwise fashion while the airway was intermittently occluded at the mouth for periods of one to two seconds. During each occlusion, measurements of pressure and of change in lung volume were made. Approximately five deflations were usually performed. Values of pressure and absolute lung volume were plotted against total lung capacity and functional residual capacity. Discrepant curves were ignored.



Figure—In this lung model, the lung is idealized as a diaphragm with area equal to unity moving in a cylinder.

For each of the 14 patients the constants a, b, and c were determined by an iterative computer-assisted process so as to give the best least squares fit (10) of the pressure-volume pairs to Eq (1). The degree to which the model fit the data was assessed by the proportion of variance in the data accounted for by the model (10).

The parameters R, E, $F'(0)$, and E*FRC were determined for each patient with the aid of Eq (9), (10), (12), and (13), respectively. The MSRP and CR were determined from the original pressure-volume data rather than derived from the fitted curve.

All possible paired correlations were calculated among $F'(0)$, E*FRC, MSRP, CR, E, R, and $-c$. Student's *t* test (10) was used to determine statistical significance.

Results

Patient data are shown in Table 1. Patients 1 through 7 represented a group with decreased compliance at FRC; patients 8 through 14 represented a group with increased compliance. For all but three patients the model accounted for over 99% of the data; for all patients this value was greater than 96%.

Correlations among the measures $F'(0)$, E*FRC, MSRP, CR, E, R, and $-c$ are presented in Table 2; statistical significance of the correlations is indicated.

E*FRC correlated closely ($r = 0.97$; $p < 0.001$) with $F'(0)$. This is not surprising since they represent the slopes of the pressure-volume curve at two closely related points. The statistically significant correlation between E*FRC and R ($r = 0.67$; $p < 0.01$) might also have been expected given that, as shown by Eq (13), they are equal when $V(FRC) = V(MAX) - 1$. E*FRC was not closely associated with the other measures, although a statistically significant relationship was observed between E*FRC and CR ($r = 0.52$; $p < 0.05$). MSRP and CR were closely associated with each other ($r = 0.85$; $p < 0.001$) but not with the other measures. The close but inverse relationship between $-c$ and R is consistent with Eq (8).

Table 1
Patient Data

Subject	FVC	FEV 1	FEF 25-75	Plethysmograph TLC	MSRP	Expiratory Compliance at FRC	CR
1	4.03	2.53	0.74	6.58	26.5	0.133	4.03
2	2.67	2.28	3.56	3.59	76	0.060	21.20
3	1.66	0.81	0.28	3.45	40	0.040	11.60
4	0.79	0.45	0.19	3.44	13	0.022	3.78
5	3.89	3.35	3.72	5.21	39	0.105	7.49
6	1.05	0.78	0.52	2.30	19	0.036	8.26
7	1.08	0.72	0.43	3.16	34	0.041	10.80
8	4.19	3.02	1.99	5.91	39	0.250	6.59
9	3.54	2.78	2.47	4.56	52	0.200	11.40
10	4.69	3.99	4.87	6.93	19	0.320	2.74
11	4.81	4.21	5.39	6.15	51	0.222	8.10
12	1.94	1.53	1.46	3.51	21	0.172	6.25
13	2.87	2.48	3.25	3.81	62	0.230	16.30
14	7.35	5.54	4.04	8.75	39	0.470	4.46
Subject	**F′(0)**	**E*FRC**	**E**	**R**	**a**	**b**	**c × 100**
1	4.22	5.78	16.90	14.00	6.68	−3.33	−7.13
2	11.30	14.90	20.80	25.60	3.75	−2.26	−3.91
3	4.94	8.78	1.39	6.19	3.40	−1.25	−16.20
4	7.72	7.83	−2.42	4.51	3.48	−0.58	−22.20
5	3.05	4.60	14.20	11.00	5.33	−3.61	−9.07
6	10.10	10.60	−2.11	7.66	2.27	−0.76	−13.10
7	17.70	19.20	1.12	18.80	3.45	−1.06	−5.32
8	0.62	2.41	16.50	6.86	5.95	−11.10	−14.60
9	1.07	2.27	11.80	6.52	4.54	−6.09	−15.30
10	1.93	2.56	16.60	10.00	7.82	−5.20	−9.96
11	2.28	2.73	14.60	9.93	6.32	−4.35	−10.10
12	0.25	2.23	11.50	4.12	3.67	−16.30	−24.30
13	2.54	6.05	16.40	11.10	3.80	−4.38	−8.98
14	1.04	1.66	18.60	8.73	8.79	−8.40	−11.40

Patients 1 through 7 represent a group with decreased compliance at FRC; patients 8 through 14 represent a group with increased compliance.

TLC = total lung capacity, MSRP = maximal static recoil pressure, CR = coefficient of retraction, and E*FRC = elasticity at FRC.

Table 2
Correlations Among Measured and Derived Parameters of Lung Elasticity

	F′(0)	E*FRC	MSRP	CR	E	R
E*FRC	0.97†					
MSRP	0.01	0.15				
CR	0.37	0.52‡	0.85†			
E	−0.52§	−0.45	0.53§	0.14		
R	0.61‡	0.67‡	0.56§	0.60§	0.34	
−c	−0.40	−0.43	−0.52§	−0.41	−0.42	−0.84†

†$p < 0.001$.
‡$p < 0.01$.
§$p < 0.05$.
E*FRC = elasticity at functional residual capacity, MSRP = maximal static recoil pressure, and CR = coefficient of retraction.

Discussion

Our observations concur with earlier reports that the relationship between lung pressures and lung volumes can be closely approximated by the exponential function described by Eq (1). Although we were able to construct a model lung with this same pressure-volume relationship, the model clearly does not describe real lung mechanics and therefore does not provide an explanation as to why this relationship is exponential.

In the case of an ideal spring, the elastic force at any moment is a function of the spring's "force constant" and the distance the spring has been stretched. Hence, the force constant provides a description of the elasticity of the spring. In the case of the model lung, the elastic force is dependent upon both R and E as well as on the position of the diaphragm. Therefore, a single parameter is inadequate to describe the elasticity of the model lung. This does not imply that the elasticity of a real lung cannot be described by a single parameter, but it does suggest that this need not be the case.

How the elastic components of the lung are structurally integrated to function in the manner described by Eq (1) is not well understood. It is not surprising, therefore, that a disparate array of measures have been introduced in an attempt to describe "lung elasticity." This point is well illustrated by the lack of a close relationship between E*FRC and MSRP or CR, since these all have been regarded as measures of lung elasticity. A more satisfactory situation awaits a better understanding of the relationship between lung structure and lung function.

References

1. Salazar E, Knowles J. An analysis of pressure-volume characteristics of the lungs. J Appl Physiol 1964;19:97-104.

2. Gibson GJ, Pride NB, Davis J, Schroter R. Exponential description of the static pressure-volume curve of normal and diseased lungs. Am Rev Respir Dis 1979;120:799-811.

3. Turner J, Mead J, Wohl M. Elasticity of human lungs in relation to age. J Appl Physiol 1968;25:664-71.

4. Pengelly LD. Curve-fitting analysis of pressure-volume characteristics of the lungs. J Appl Physiol 1977;42:111-6.

5. Fishman AP (ed). Pulmonary disease and disorders. New York: McGraw-Hill, 1980.

6. Marshall R. The physical properties of the lungs in relation to the subdivisions of lung volume. Clin Sci 1957;16:507.

7. DuBois AB, Botelho SY, Bedell GN, Marshall R, Cornrol JH. A rapid plethysmographic method for measuring thoracic gas volume: A comparison with a nitrogen washout method for measuring functional residual capacity in normal subjects. J Clin Invest 1956;35:322-6.

8. Gibson GJ, Pride NB. Lung distensibility: The static pressure-volume curve of the lungs and its use in clinical assessment. Br J Dis Chest 1976; 70:143-84.

9. Mead J. Volume displacement body plethysmography for respiratory measurements in human subjects. J Appl Physiol 1960;15:736-40.

10. Hays W (ed). Statistics for the social sciences. New York: Holt, Rinehart, and Winston, Inc, 1973:616-74.